[3.5] Prepare a system in direction  and make a measurement in direction . Show that  where  is the angle between .

Solution. In spherical coordinates

 and . (1)

From Problem 3.4 the eigenvalues and eigenvectors for  are

.

For  they are

.

Since  we have

 (2)

Recall the Half-Angle Formula for cosine:

 (3)

So

 (4)

The system is prepared in the direction  so  is the state vector. We measure in direction  so  is the observable (Hermitian operator). Thus

 (5)

Since ,

 (6)

Since ,

 (7)

To simplify the multiplication of (6) and (7) we require several trig identities.

DeMoivre's Theorem () can be used to easily show

 (8)

The Half-Angle Formula for sine is

 (9)

The Double-Angle Formula for sine is

 (10)

The cosine formula for Difference of Two Angles is

 (11)

Finally







